Ling 320. Lecture Notes. 18 September 2007.

1. Announcements.

-Change in schedule: The first exam will be held in class Tuesday, October 2nd.
-My office hours are from 4:15-4:45, T/Th, in H-663-5.

2. Another Presupposition Trigger: The Cleft Construction.

The sentence in (3) illustrates the *Cleft Construction*:

(1) It was Keir who invited Michael.

This construction has the general form in (2).

(2) It was *X* who *Y*.

The cleft construction contrasts with simple declarative sentences, such as (3):

(3) Keir invited Michael.

Clefts differ from regular declarative sentences in a semantically interesting way: cleft constructions are presupposition triggers, while regular declaratives are not. In particular, while both (1) and (3) entail (4), only (1) additionally presupposes (4).

(4) Someone invited Michael.

That (3) entails (4) is illustrated by the following sentence, which shows that stating (3) but denying (4) is contradictory:

(5) Keir invited Michael, but it's not the case that someone invited Michael.

(3) however does not presuppose (4), as (4) does not project:

(6) a. Keir didn't invite Michael. -/-> Someone invited Michael.
b. Maybe Keir invited Michael. -/-> Someone invited Michael.
c. If Keir invited Michael, we're set. -/-> Someone invited Michael.

In contrast, (1) both entails (4), as (7) shows, and presupposes (4), as the examples in (8) show.

(7) It was Keir who invited Michael, but it's not the case that someone invited Michael.

(8) a. It wasn't Keir who invited Michael. → Someone invited Michael.
b. Maybe it was Keir who invited Michael. → Someone invited Michael
c. If it was K. who invited M., we're set. → Someone invited Michael.

Exercise: What does the following cleft sentence presuppose?

(9) It was Michael who Keir invited.

3. Where we're at, and where we're going

Our goal
A theory of semantic competence.

Observations so far
Native speakers have intuitions about what kind of inference relations hold between sentences. They know (at least implicitly) whether a sentence entails, implicates, presupposes, or is in no relation to another sentence.

Sentence meaning
How can we represent what native speakers know about sentence meaning?

Question
How do we understand sentences we have never heard before?

Principle of Compositionality (Frege)
The meaning of a sentence is computed from the meanings of its parts, and the way those parts are assembled syntactically.

Immediate goal
Explore how the meaning of a sentence is computed from the meanings of its parts.

How to reach that goal
Posit suitable set theoretic entities as meanings for sentence parts.

4. Set Theory

A *set* is a collection of objects of any kind.
E.g., the set of students in Ling 320, the set of letters in the English alphabet

Objects in a set are called the *members* or *elements* of that set.
E.g., Tro is a member of the set of students in Ling 320

A set can be formed out of elements that have no connection whatsoever.
E.g., the set consisting of you, the word *me*, and the square root of 7

Sets may have any number of elements, finite or infinite.
E.g., the set of desks in this room, the set of sentences of English

A set with only one member is called a *singleton* set.
E.g., the set with only you in it

Sets themselves may be members of other sets.
E.g., the set of all sets consisting of you and a planet

There is one set with no members at all, the *empty set* or *null set*, symbolized as $\varnothing$.

Some notational conventions

We use italic capital letters (A, B, C…) to refer to sets and italic lower-case letters (a, b, c…) to refer to the individual objects that are members of the sets.

We use $\in$ to symbolize the membership relation, so that $b \in A$ is read as 'b is a member of A'. If we want to say that a is not a member of B, we write $a \notin B$.

Since sets may be members of other sets, we will sometimes write $A \in B$ when the set A is a member of the set B.

4.1 Set specification. There are at least two ways to specify a set:

(i) by listing all its members (*list notation* or *enumeration*);

(ii) by stating a property that an object must have to be a member of the set (*predicate notation* or *description*).

List notation. To specify a set by list notation, the names of the members, separated by commas, are enclosed in braces. E.g., {Tro, Kumiko, Melissa}.

Predicate notation. To specify a set by *predicate notation*, we specify a property that the members of the set share.

E.g., $\{x \mid x$ is a sentence of Tagalog$\}$
$\{x \mid x$ is a student in this class whose first name starts with $K\}$

The vertical line after the first occurrence of the variable x is read as 'such that'. These expressions read as 'the set of all x such that x is a sentence of Tagalog', and 'the set of all x such that x is a student in this class whose first name starts with K'.

4.2 Relations between sets

Identity. Two sets are *identical* if and only if ('iff') they have exactly the same members. We symbolize identity by '='.

E.g., {x | x is a student in this class whose first name starts with K} =
{Kumiko, Kris, Kevin, Kinneret}

Because these two sets pick out the same members, we say they are the same set.

Note that as a consequence of this definition of identity:

(i) Order does not matter.
E.g., {Tro, Melissa, Kumiko} = {Kumiko, Melissa, Tro}.

(ii) The number of times an object is listed does not matter.
E.g., {Tro, Tro, Tro, Kumiko, Melissa} = {Tro, Kumiko, Melissa}.

Practice 1

True or False:

a. $b \in \{b, c\}$ T   b. $c \in \{b, c\}$ T   c. $\{c\} \in \{b, c\}$ F   d. $\{b\} \in \{b, c\}$ F
e. $b \in \{b, \{c\}\}$ T   f. $c \in \{b, \{c\}\}$ F   g. $\{c\} \in \{b, \{c\}\}$ T h. $\{b\} \in \{b, \{c\}\}$ F
i. $\{b, c\} = \{c, b\}$ T j. $\{\text{Tro}\} \in \{\{\text{Tro}\}\}$ T   k. Tro $\notin \{\{\text{Tro}\}\}$ T

Subsets

A is a subset of B (symbolized as $A \subseteq B$) iff every member of A is a member of B. Note that the subset relation allows any set to be a subset of itself.

Examples: $\{a, b\} \subseteq \{a, b, c\}$
$\{a\} \subseteq \{a, b, c\}$
$\{a, b, c\} \subseteq \{a, b, c\}$

Note that the null set is a subset of every set. That is, for any set A, $\varnothing \subseteq A$. The reasoning is as follows: Since $\varnothing$ has no members, the requirement that every member of $\varnothing$ is a member of A holds, even if vacuously. Similarly, for $\varnothing$ to not be a subset of A, there would have to be some member of $\varnothing$ that was not also a member of A. This is impossible since $\varnothing$ has no members.

A is a *proper subset* of B (symbolized as $A \subset B$) iff every member of A is a member of B, and there is at least one member of B that is not a member of A.

Examples: $\{a, b\} \subset \{a, b, c\}$
$\{a\} \subset \{a, b, c\}$
$\{a, b, c\} \not\subset \{a, b, c\}$

Practice 2

True or false:

a. {Tro} $\subseteq$ {Tro, Kumiko, Melissa} b. {{Tro}} $\subseteq$ {Tro, Kumiko, Melissa}

Assume that $A = \{b, \{c\}\}$. True or false:

c. $b \in A$ T d. $\{b\} \subseteq A$ T e. $\{c\} \in A$ T f. $\{c\} \subseteq A$ F

g. $\{b, \{c\}\} \subseteq A$ T h. $c \in A$ F i. $\{\{c\}\} \subset A$ T j. $\{b, \{c\}\} \not\subset A$ T

k. $\{b, \{c\}\} \in A$ F l. $\varnothing \in A$ F m. $\varnothing \in \{\varnothing\}$ T n. $\varnothing \subseteq A$ T

4.3 Operations on sets

The number of members in a set A is called the *cardinality of A*, written $|A|$.

E.g., |{Kumiko, Kris, Kevin, Kinneret}| = 4 = |{a, b, c, d}|

The *union* of two sets A and B, written $A \cup B$, is the set whose members are just the objects that are members of A or B or both.

(10) $A \cup B =_{\text{def}} \{x \mid x \in A \text{ or } x \in B\}$

Note that the disjunction 'or' allows an object to be a member of both A and B.

Examples: $\{a, b, c\} \cup \{a, b, \text{Tro}\} = \{a, b, c, \text{Tro}\}$
$\{a, b, c\} \cup \{\{c\}, d\} = \{a, b, c, \{c\}, d\}$
$\{\text{Tro}\} \cup \{\text{Tro}, \{\text{Tro}\}\} = \{\text{Tro}, \{\text{Tro}\}\}$
$\{a, b, c\} \cup \varnothing = \{a, b, c\}$

The *intersection* of two sets A and B, written $A \cap B$, is the set whose members are just the members of *both* A and B. This operation is called the intersection of A and B, written as $A \cap B$.

(11) $A \cap B =_{\text{def}} \{x \mid x \in A \text{ and } x \in B\}$

Examples: $\{a, b, c\} \cap \{a, b, \text{Tro}\} = \{a, b\}$
$\{a, b, c\} \cap \{\{\text{c}\}, \text{d}\} = \varnothing$
$\{\text{Tro}\} \cap \{\text{Tro}, \{\text{Tro}\}\} = \{\text{Tro}\}$
$\{a, b, c\} \cap \varnothing = \varnothing$

The *difference* between two sets, written $A - B$, 'subtracts' from A all objects in B.

(12) $A - B =_{\text{def}} \{x \mid x \in A \text{ and } x \notin B\}$
Examples: $\{a, b, \text{Tro}\} - \{\text{Tro}\} = \{a, b\}$
$\{a, b, \text{Tro}\} - \{\text{Tro}, \{\text{Tro}\}\} = \{a, b\}$
$\{a, b, \text{Tro}\} - \{\{\text{b}\}\} = \{a, b, \textit{Tro}\}$
$\{a, b, \text{Tro}\} - \{a, b, \text{Tro}\} = \varnothing$

The *complement* of a set A, written A', is the set consisting of everything not in A.

(13) $A' =_{def} \{x \mid x \notin A\}$

Note: Every statement involving sets is made against a background of assumed objects, which comprise the *universe of discourse* for that discussion, conventionally represented as U. Thus, (13) is equivalent to:

(14) $A' = U - A$

Examples: Assume that U = {x | x is in Ling 320} ∪ {Charles}

{x | x is in Ling 320}' = {Charles}
(∅')' = ∅
({x | x is in Ling 320} ∪ {Charles})' = ∅
{x | x's name is longer than three letters}' = {Tro, Jon}

Sometimes we need to refer to the set whose members are all the subsets of a given set A. This set is called the *power set* of A, written as $\wp(A)$.

E.g., $\wp(\{a, b\}) = \{\{a, b\}, \{a\}, \{b\}, \varnothing\}$

Example: ℘({London, Edinburgh, Dublin}) =
{{Longdon, Edinburgh, Dublin}, {London, Edinburgh}, {London, Dublin}, {Edinburgh, Dublin}, {London}, {Edinburgh}, {Dublin}, ∅}

5. Environments that reverse entailment patterns.

[Note: This material is from past lectures, added here so that you have it in writing].

We have observed four linguistic environments in which entailment patterns are reversed: negated sentences, *if*-clauses, *every*-relatives, and *any*-relatives.

For example, in a positive sentence, (1a) entails (1b).

(15) *Positive sentences*

a. Sara is a little dog.
b. Sara is a dog.

When the same two sentences are negated, this entailment pattern is reversed, so that (2a) entails (2b).

(16) *Negated sentences*

a. Sara is not a dog.
b. Sara is not a little dog.

Entailment patterns are also reversed in *if*-clauses, as (3a) entails (3b).

(17) If-*clauses*

a. If Sara is dog, she can't get on the plane.
b. If Sara is a little dog, she can't get on the plane.

Entailment patterns are also reversed in '*every* relative clauses' and '*any* relative clauses'. A *relative clause*, roughly speaking, is a clause that modifies a noun. For instance, in the example *every person that you talked to*, '*that you talked to*' is a relative; it is a clause that modifies *person*.

In the following examples, the (a) sentences entail the (b) sentences, demonstrating that entailment patterns are reversed in these cases as well.

(18) Every-*relatives*

a. Everyone who has a dog can't get on the plane.
b. Everyone who has a little dog can't get on the plane.

(19) Any-*relatives*

a. Anyone who has dog can't get on the plane.
b. Anyone who has a little dog can't get on the plane.