the end of Lecture I, you will see that we are skipping a step in Montague's interpretation of a fragment of English. What Montague did in *PTQ* was not interpret "disambiguated English" directly, but rather define another language of intensional logic, IL, give rules for translating English into this language and then give an interpretation for IL. The type hierarchy is built right into the structure of IL. The important thing here is that this hierarchy imposes a certain structure on the denotations assigned—indirectly—to the English expressions given in his grammar. Montague took over the type hierarchy from earlier work by Russell, Church, and others.)

Early extensions of Montague's work often used this hierarchical kind of universe to deal with more kinds of expressions than Montague had included in his fragment. For example, suppose we want to understand sentences such as these:

11. That proposition is false.
12. The fact that it is raining does not bother me.

Apparently, some expressions of English refer to things like propositions and it is natural to think that a noun phrase like *the fact that it is raining* should get the same kind of interpretation as a clause like *that it is raining*. The latter kind of clause was included in *PTQ* and was interpreted as a proposition. Often we find that verbs that take *that*-clauses as objects can also take "propositional" noun phrases as objects:

13. I do not believe that the earth is flat.
14. I do not believe that proposition.

So a natural thing to do (and it was done in Delacruz, 1976) is to let nouns such as *proposition* denote sets of propositions. Thus, they have an entirely different TYPE of denotation. Similarly, if we ask how we are to understand plural common nouns such as *dogs*, that we should think of them as denoting SETS of entities seems natural. We have the apparatus for doing this right in Montague's hierarchy of types. And that is just what Michael Bennett (1974) did in the first serious study of plurals in English within Montague grammar.

But this approach has a problem. To explain this problem, I have to say something abut Montague's theory of the relationship between syntactic categories and the kinds of denotations that go with them. In his theory, part of the interpretation of a formal language is a function that maps syntactic categories into types of denotations. This means that if two words in the formal language belong to the same syntactic category, then they cannot have denotations of different types. Thus, in interpreting English in the way I have just outlined, the common nouns *dog*, *dogs*, and *proposition*, have to belong to different syntactic categories. This makes me uncomfortable as a linguist, because it makes it a complete accident that the syntax of noun phrases, for example, for these different kinds of common nouns should have anything in common.

Let me stop, for now, with just this hint at why a difficulty arises when using Montague's type theory for getting different kinds of denotations for words that apparently belong to the same syntactic category. For the rest of this lecture, I will explore another sort of approach to the undoubted differences of meaning that words of the same syntactic class seem to exhibit. This second kind of approach takes the position that we can think of E—the domain of individuals—as being structured in some way. I will start with a simple example and then spend the rest of the lecture going into more detail on two topics: plurals and terms for kinds.

Consider the following sentences:

15. Caesar is a prime number.
16. The theory of relativity is shiny.

They are certainly unusual sentences. Most people would say they do not make sense. What are we to say about them? We could say that they are just false. But given our way of looking at meanings, we might want to ask if there could be a possible world in which they were true. My reaction is to say "No." And if I consult my intuitions, I would probably say something like this: the reason that these sentences are unusual is that Caesar (a man) isn't the KIND of thing that can be a prime number and the theory of relativity isn't the SORT of thing that can be shiny (or not shiny).

I am now going to be talking about approaches that quite literally, incorporate such ideas, that is, theories in which the domain E is split into different sorts of things, and we take this difference into account when we build up complex expressions involving words that correspond to different sorts. So the basic structure of our model *(M2)* remains the same, but E is given some internal structure. For the examples just given, we could say that E is the union of two subdomains $E1$ and $E2$, say, corresponding to physical objects such as people and cars (which can be shiny) and abstract objects like theories and numbers (which can never be shiny).

Let us think, now, about the meaning of plurals. As you know, English makes an obligatory distinction between singular common nouns *(horse)* and plurals *(horses)*. What are the meaning differences, if any, between these two sorts of expressions? I want to concentrate especially on a particular theory of plurals proposed a few years ago by the German linguist Godehard Link. But I would like to put this question into a slightly larger context, namely that of asking about the relationships between meanings in one language and meanings in another language, particularly with regard to the nature of grammatical or obligatory distinctions and their semantics. This larger question is one that should be raised, but it has not been touched on very much in the tradition of formal semantics that we are mostly dealing with in these lectures. The phenomenon of a required distinction of singular and plural, or singular, dual, and plural is certainly not a universal one. And the question is this: what is the relationship between the meanings of words such as *horse* and *horses* and the meaning of a word like *mǎ* in Chinese, which does not have this obligatory distinction.

This sort of question is extremely important if we are interested in pursuing problems of general linguistic theory in the domain of semantics. In this general line of thought, we are trying to answer the important big question: What is Language? We are also trying to find answers to equally important questions: How do and how can languages differ? These two questions are, in a sense, merely statements of what linguistics is all about.

Some linguists and other thinkers about language seem to take the line that semantics should be or is "more universal" than other parts of linguistic theory. One might want to make this claim for two reasons: One is to say that meanings are determined by thought, the categories of thinking that we all share as humans; the other is to say that meanings are determined by the way the world is, and we all inhabit the same world. There is reason to be skeptical about both of these reasons, and many writers have taken quite opposite views, one in particular which (stated very crudely) says that language is a kind of screen through which we see the world and with which we think. Languages differ in obvious ways. Therefore, we should expect the meanings of words and other expression in different languages to differ quite markedly. While we cannot make *a priori* arguments about these matters, we can propose and test hypotheses about universals of meaning in the same way as is done in phonology or syntax.

To introduce Link's theory of plurals, let me reiterate that Montague's analysis of English dealt only with singulars: singular noun phrases, singular nouns, verbs that go with them (*walks* and not the inflected forms for plurals like *walk, are,* and so on). The denotation Montague assigned to a singular common noun such as *horse,* was just what we did in our interpretation of PC: *D*(horse) = the set of horses (It differed at the bottom level in the way I indicated earlier in this lecture due to the use of individual concepts.)

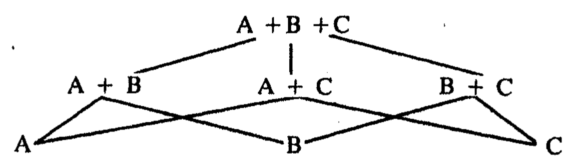
I assume you all know what a horse is. What is the denotation of *horses* used as a common noun as in this sentence:

17. These horses are white.

Michael Bennett gave the first answer to this question in Montague-style semantics. Bennett said: *D(horses)* is the set of sets of horses, so that every time we use the word *horses,* we are referring to sets of horses. Suppose we have an individual horse, call him Abraham, then he would fall within the denotation of *horse*. But if we have two horses, Abraham and Bessie, the set consisting of the two of them would fall within the denotation of the plural *horses*.

I am not going to give a lot of arguments against Bennett's idea, but merely present Link's theory as it is—a different way of looking at plurals—because it is a good illustration of "giving more structure to the domain." (One problem with Bennett's theory is this: because the denotations of singulars and plurals is type-theoretically distinct, in the way I outlined before, all of the syntactic categories have to be duplicated, and the predicate *runs* for a singular subject has to be different from the predicate *run* with a plural subject and it is difficult to understand how we can say things such as *Abraham runs and so do those other horses*. I will return to this problem of "syntactic inflation" in the next lecture.)

Link said that every common noun like *horse* has associated with it a certain kind of structure that we get by allowing an operation for forming what he called plural individuals. So we can take Abraham and Bessie and form a plural individual (Abraham + Bessie), which might be the meaning of *Abraham and Bessie* or *those two horses*. And we can then define a certain part-whole relationship such that Abraham is part of the plural individual (Abraham + Bessie). How is this different from Bennett's way? The crucial difference is that these plural individuals are strictly elements in *E,* the domain of individuals. The following illustration shows what such a structure looks like: If we have three horses in the domain, Abraham (A), Bessie (B), and Clara (C), then the substructure of horses in *E* would look like this:



The lines indicate the part-whole relationship I mentioned and "+" (as I used it already) indicates the operation of forming a plural individual—or "fusion"—of two individuals that can continue. Note, however, that the operation is stipulated to be associative—that is, (A + B) + C = A + (B + C)—which is why we are justified in dropping the parentheses and just writing one thing on the top line. The items at the bottom are called *atoms,* they are the things that have no strictly smaller individual parts.

To repeat, Link says the denotations of *horses* is just the set of plural individuals that are horses. Actually, Link did not say that exactly, In order to have a nice algebraic structure, we need to have the atoms at the bottom as part of the substructure in the domain; to give the exact meaning of the English common noun *horses,* however, Link had to exclude the atoms, so the real meaning of *horses* is the set of plural individuals that are horses minus the atoms. And that is something specific about English that has to be said: the denotation of *horse* is just the atoms, that of *horses* just the nonatoms. Every common noun, better, every common count noun in English has associated with it an algebraic structure of this sort. And in English, just as here, we have to exclude the atoms from the denotation of the plural common noun.

Now what might we say about a word such as *mǎ* in Chinese? Suppose we assume that Chinese has basically the same kind of structure associated with each common count noun such as *mǎ*. The we could say that the only difference between Chinese and English in this regard is that for Chinese one does not have to make this special distinction between atoms and nonatoms in the structure. If we are correct, the denotation of *mǎ* is just the union of the denotations of *horse* and *horses*.

$$D(\text{mǎ}) = D(\text{horse}) \cup D(\text{horses})$$

This seems to me to be a rather satisfying way of looking at the difference between Chinese and English in this particular case. And it seems to me that quite a lot of differences between languages with regard to distinctions required in one language but not in another might turn out to be something like this. Notice that there is no sense in which it is claimed that *mǎ* is ambiguous.

I want to be a bit speculative and say something about another striking difference between English (and many other languages), on the one hand, and Chinese (and many other languages), on the other hand: the way in which we count. One of the first things that an English speaker has to learn about Chinese (Japanese, Thai, and so on) is that one cannot merely take a number and a common noun and put them together as we do in English (*three horses, five tables, six pigs*). You have to learn a whole system of *classifiers* or *counters* and put together the right kind of counter and the number word to get the equivalent expressions. So, for example, the classifier for *horse* in Chinese is *pǐ*. Now here's the speculation: perhaps the lack of an obligatory singular-plural distinction and the use of classifiers of this sort go together and might be explained in the following way. Referring to our diagram of the algebraic structure associated with *mǎ* or *horse* and *horses*, we might ask: How many *mǎ* are there? It depends on how you count! If we allow plural individuals that overlap to count as different horses, then there are seven; if we exclude these, then we get a number of possible answers: one (A + B + C), or three (just the atoms), or two (in three different ways: A and B + C; or B and A + C; or C and A + B)—All of which can be very confusing! Suppose the meaning of the classifier is a way of introducing explicitly the difference between the denotations of the singular and plural in English, that is, it restricts the denotation of the common noun to just the atoms: then we

have a clear, unambiguous answer. If this is right, then a distinction built into the obligatory distinction in English, gets built back into Chinese, where it counts.

Link's paper treats another extremely interesting topic, the semantics of mass nouns like *mud, blood,* and so on. Briefly, his method is to introduce some more structure into the domain. There is a special subset of atoms in the domain that I will call *S* (for "stuff"; Link calls it *D*); the elements of this subdomain are to be thought of as "quantities of matter" and they participate in their own algebraic structure, with their own part-whole relation. Unlike the algebra of count nouns, however, the algebra of stuff is not atomic; that is, there are not necessarily any smallest chunks of matter. (Admittedly, this is confusing because I said they are atoms, but that is within the general other algebraic structure for *E* as a whole.) Moreover, this domain of stuff is related to the big domain *E* in a special way: there is a mapping from *E* to *S* that preserves part-whole relation in such a way that the stuff corresponding to the individual parts of a fusion of individuals in *E* must comprise parts of the stuff corresponding to the whole fusion. In English, mass nouns act more or less like all nouns in Chinese: you cannot count them directly. Again, this raises interesting cross-linguistic questions about semantics.

For the remainder of this lecture, I will consider another use or meaning for English words such as *horses,* when used as full noun phrases, as in sentence 18.

18. Horses are mammals.

I will refer especially to Greg Carlson, whose work on such *generic plurals,* as we may call them, was one of the earliest studies within the Montague tradition that suggested a fundamental change in the model structure, again one in which the domain *E* is given more structure.

At first glance, a term phrase like *horses* seems to be ambiguous. Look at the following sentences:

19. Horses have tails.
20. Horses were galloping across the plain.

In sentences 18 and 19, we seem to be thinking about all horses:

21. All horses are mammals.
22. All horses have tails.

But in example 20 it seems as if we are talking about just some horses:

23. Some horses were galloping across the plain.

For this reason, many people have assumed that there is some kind of a hidden or "0" determiner in a noun phrase such as *horses*. But if we think about it, this seems problematical. First of all, there is what has been called the Port Royal puzzle (because it was discussed in the famous seventeenth-century work, *Logic or the Art of Thinking*, by Antoine Arnauld of the School of Port Royal). Arnauld asked what it meant to say something like example 19 or the following:

24. Dutchmen make good sailors.

It does not mean *all Dutchmen*, because we would not count the sentence as false if we found a certain Dutchman who was a terrible sailor. Carlson gives nice examples like this one:

25. Chickens lay eggs.

It is certainly false that all chickens lay eggs; only grown-up hens do. The most clinching argument against the idea that ANY determiner is found in the bare plural term phrase is the failure of such phrases to show any scope ambiguities of the sort that we noted in Lecture II with *some* and *every*. Compare these two sentences:

26. Some horse eats every kind of fodder.
27. Horses eat every kind of fodder.

Sentence 26 is ambiguous with regard to scope of the quantifiers; sentence 27 is not. In this respect, bare plurals act like names, which also do not show scope ambiguities. In short, Carlson's theory treats the denotations of bare plurals in exactly the way in which ordinary proper names are treated. But what are they names of? Once again, we want to think about different kinds of things. According to Carlson's theory, the domain E contains three different sorts of individuals: *kinds*, *objects*, and *stages*. Some sentences with the phrase *horses* are saying something about the individual kind *horse*. Others are saying something about individual instances of the kind. How do we account for the apparent ambiguity in the examples we have just been looking at? According to Carlson's theory, the ambiguity resides in other parts of the sentence. One argument for the conclusion that no ambiguity is found in the bare plural itself comes from sentences like this one:

28. Marcia hates rabbits, because they ruined her garden last spring.

Here, in the first clause, *rabbits* is understood generically, but the anaphoric *they* is used in a clause that says something about individual instances.

What is a *stage?* A *stage* is supposed to be something like a temporally/spatially limited "manifestation" of an object or kind. Let me paraphrase two sentences to illustrate the difference:

29. John smokes.
30. John is smoking.

Both sentence say something about the object-type individual John. But the second one says something like this: John is such that there is a stage of him that is smoking.

So to summarize in a much too brief way, Carlson's revision posits this kind of a sorted domain:

E = O(bjects) ∪ K(inds) ∪ S(tages)

In addition, two "realization") relations hold across these domains: *R1* relates both Kinds and Objects to Stages, *R2* relates Kinds to Objects.

Carlson's work contains much interesting material, and it has formed a starting point for lots of other work (some of which I will address in the next lecture). To end this discussion, I want to share with you an interesting puzzle from ancient Chinese philosophy. (I am indebted to Bao Zhi Ming for this example.) It has to do with the following sentence:

21. White horses are not horses.

Is this necessarily false? If we are thinking simply in terms of sets of things—white horses are things that are both horses and white—then the answer would seem to be *Yes*. But given a theory in which *white horses* involves (as a generalized quantifier) a certain Kind, a genuine philosophical (or scientific) question must be asked: What is the relation between two Kinds with different but related names: *white horses* and *horses*. Notice that, in general, we cannot assume that phrases of the form Adjective + Noun denote sets of things that come under the intersection of the denotations of the two words (fake books are not books).

We have looked very briefly at two developments in which something is done to the model structure by way of introducing different kinds of things, or *sorts* as they are often called in technical jargon. Let me end by pointing out a consequence of using sorted domains. What are we to say about sentences in which a predicate that is appropriate to one sort of thing is applied to a term of the wrong sort? One common move is to do something about the truth-value of the model structure, for example, to have three truth-values. Another, not necessarily distinct from the other, is to use not total functions, but partial ones. But that's a long story. I hope we can come back to it in our final session.